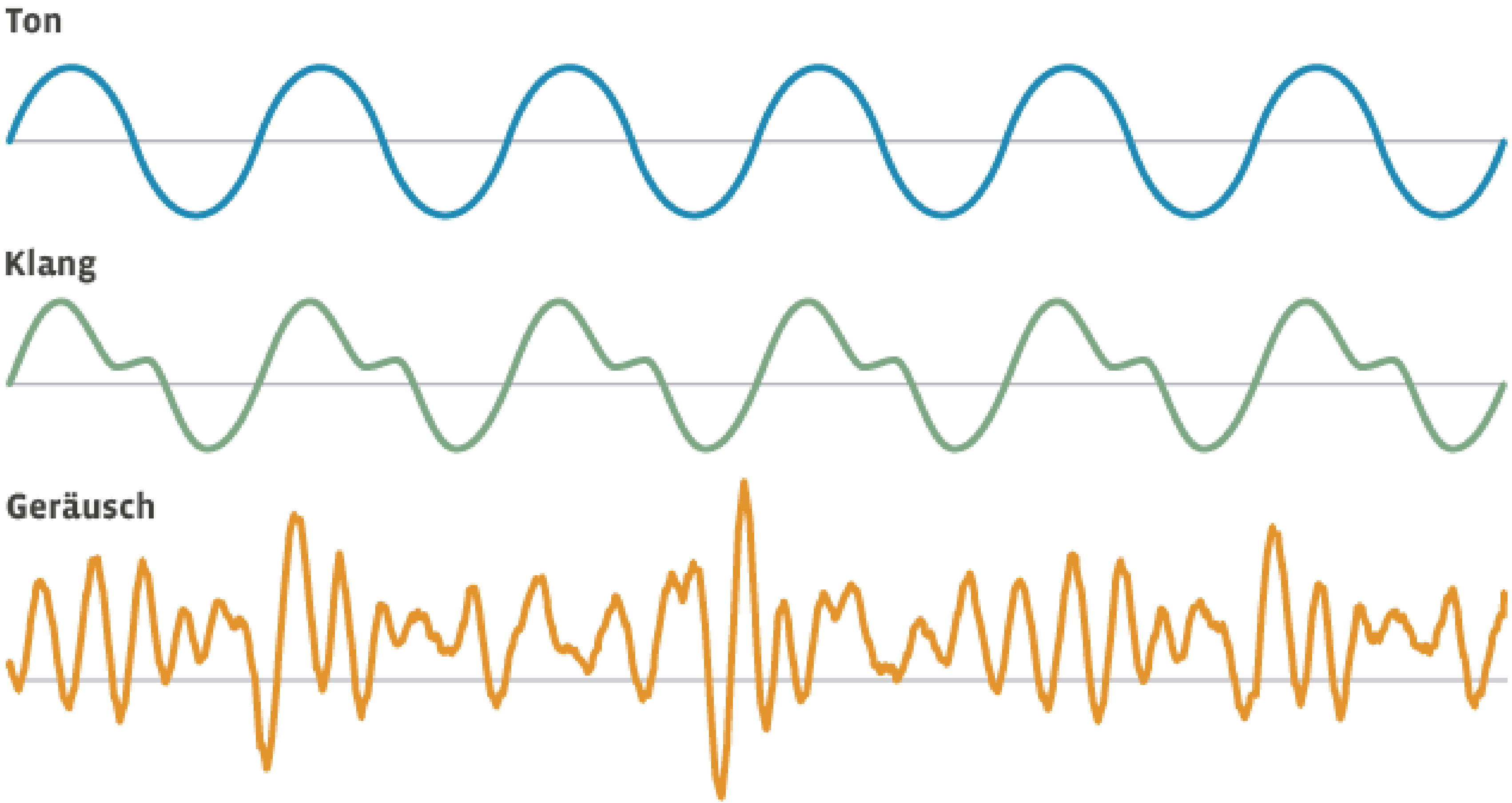
Fourierreihe

Anwendung und Darstellung

Teil 1 – Fourier Reihe

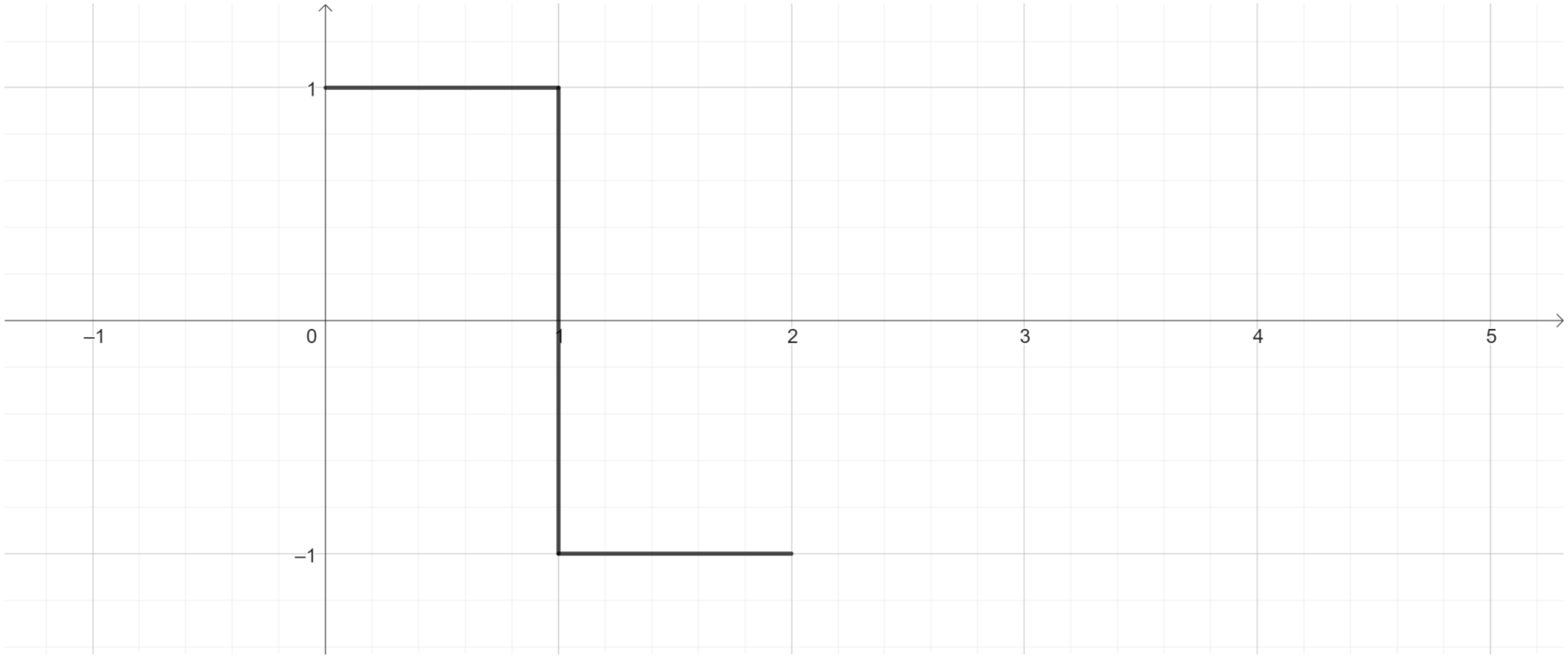
Hinführung und Aufstellen der Formel

Ziel der Facharbeit



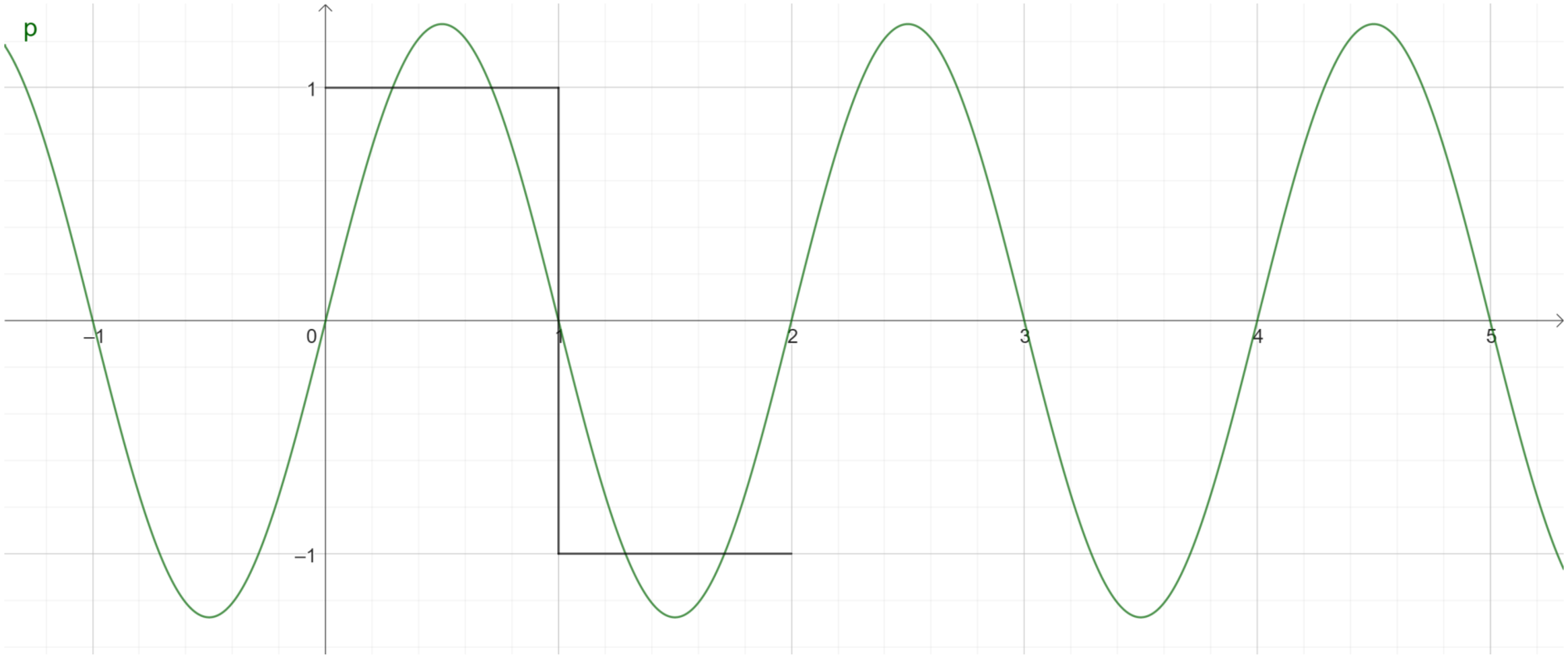
Beispiel: Eckiges Signal

Zu mathematisierendes Signal



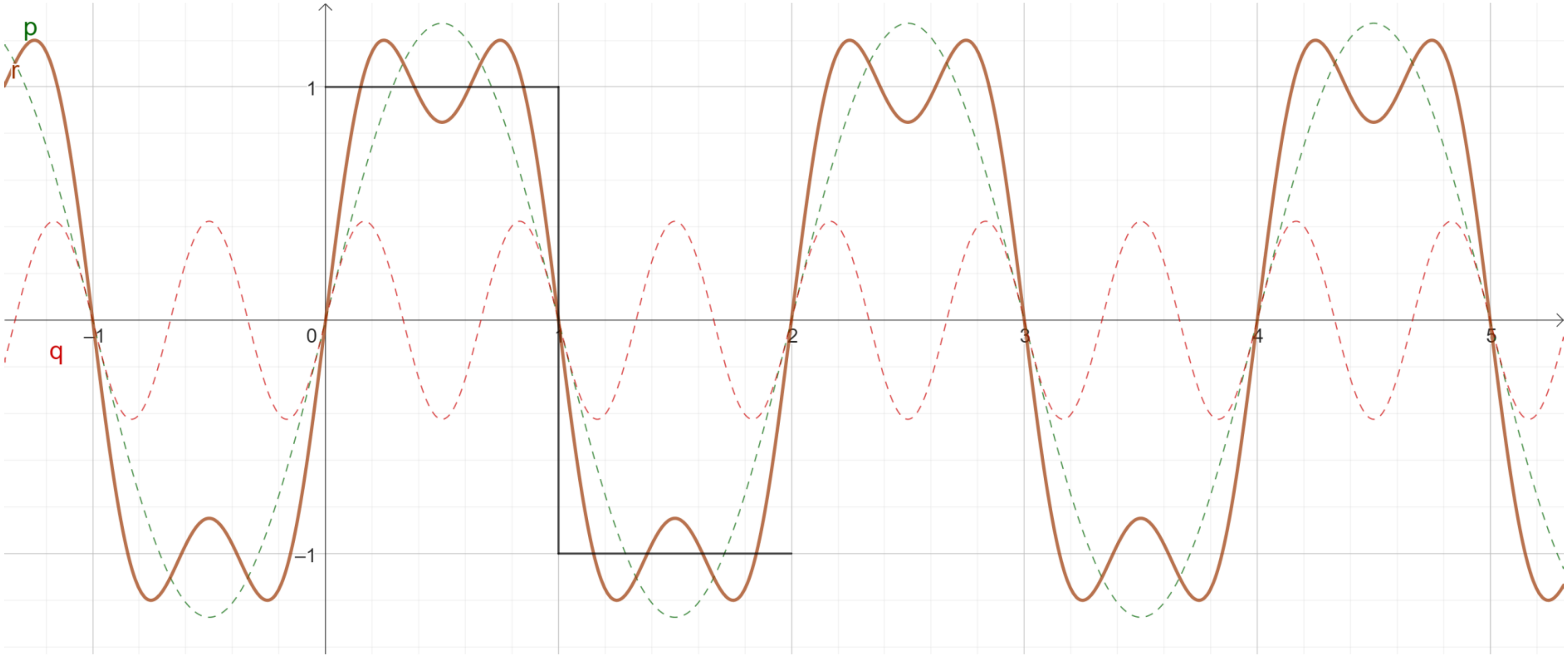
Herleitung der allgemeinen Formel über Beispiel

$$f(x) = \frac{4}{\pi} \sin(x \cdot \pi \cdot 1)$$



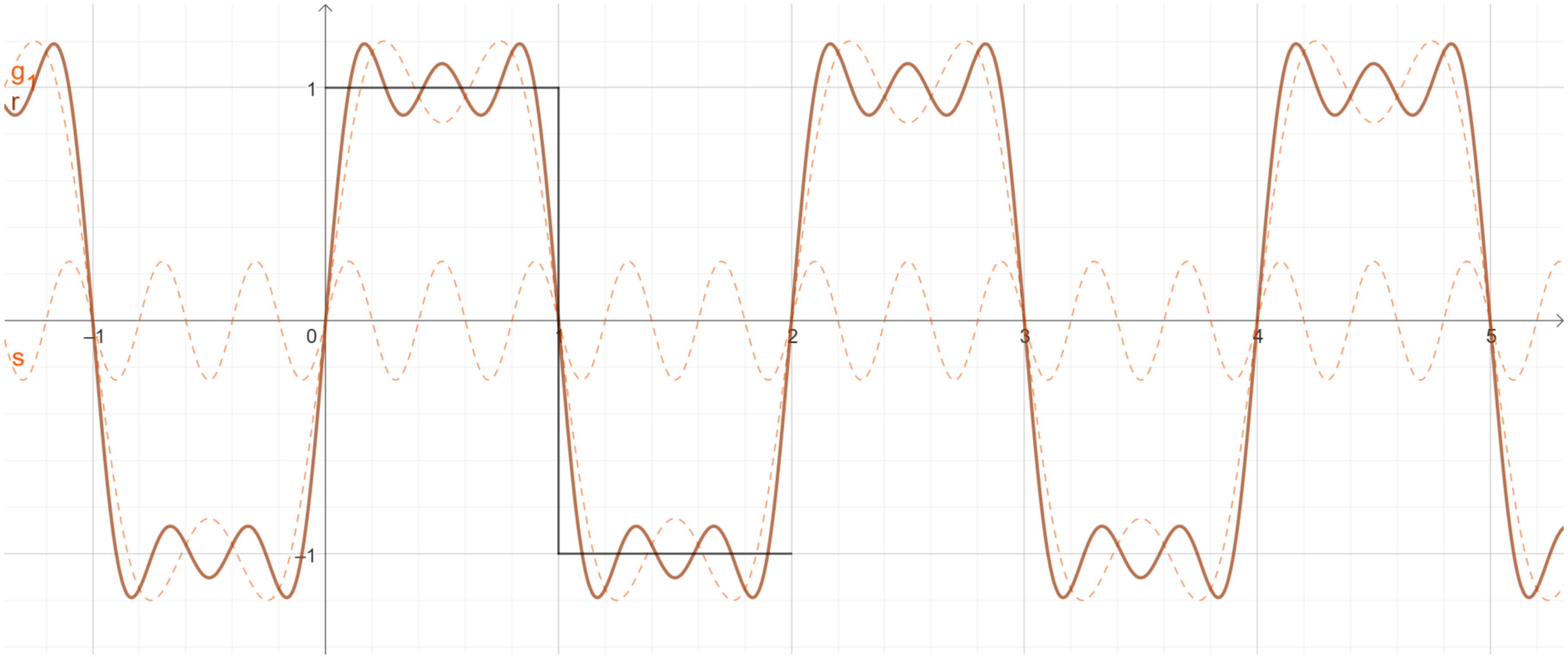
Herleitung der allgemeinen Formel über Beispiel

$$f(x) = \frac{4}{\pi}\left(sin(x \cdot \pi \cdot 1) + \boldsymbol{\frac{1}{3} sin(x \cdot \pi \cdot 3)}\right)$$



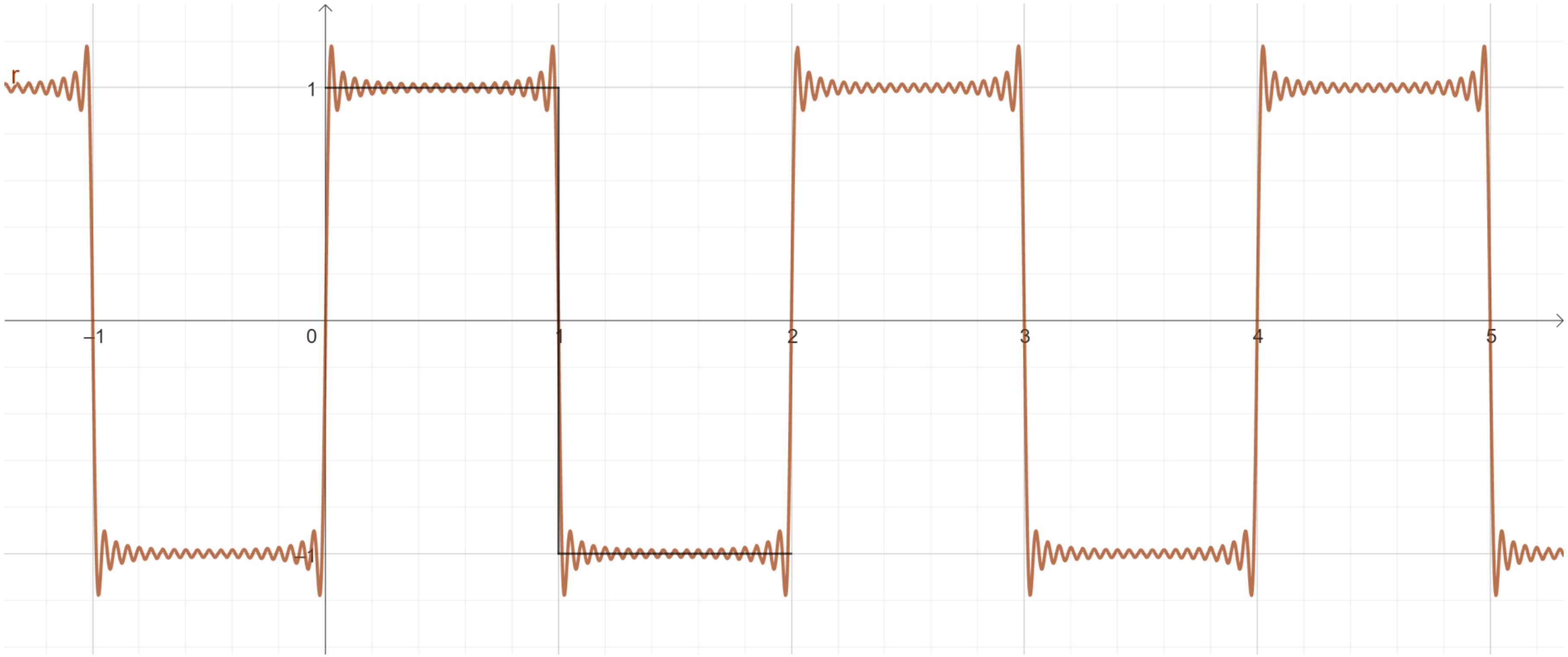
Herleitung der allgemeinen Formel über Beispiel

$$f(x) = \frac{4}{\pi}\left(sin(x \cdot \pi \cdot 1) + \frac{1}{3} sin(x \cdot \pi \cdot 3) + \boldsymbol{\frac{1}{5} sin(x \cdot \pi \cdot 5)}\right)$$



Herleitung der allgemeinen Formel über Beispiel

$$f(x) = \frac{4}{\pi}\left(sin(x \cdot \pi \cdot 1) + \frac{1}{3} sin(x \cdot \pi \cdot 3) + \frac{1}{5} sin(x \cdot \pi \cdot 5) + \ldots \boldsymbol{\frac{1}{39} sin(x \cdot \pi \cdot 39)}\right)$$



Herleitung der allgemeinen Formel über Beispiel

$$f_n(x) = \frac{4}{\pi}\sum_{k=1}^{n} \frac{\mathbf{1}}{\mathbf{2}\boldsymbol{k} - \mathbf{1}} sin\left(x \cdot \pi \cdot (\mathbf{2}\boldsymbol{k} - \mathbf{1})\right)$$

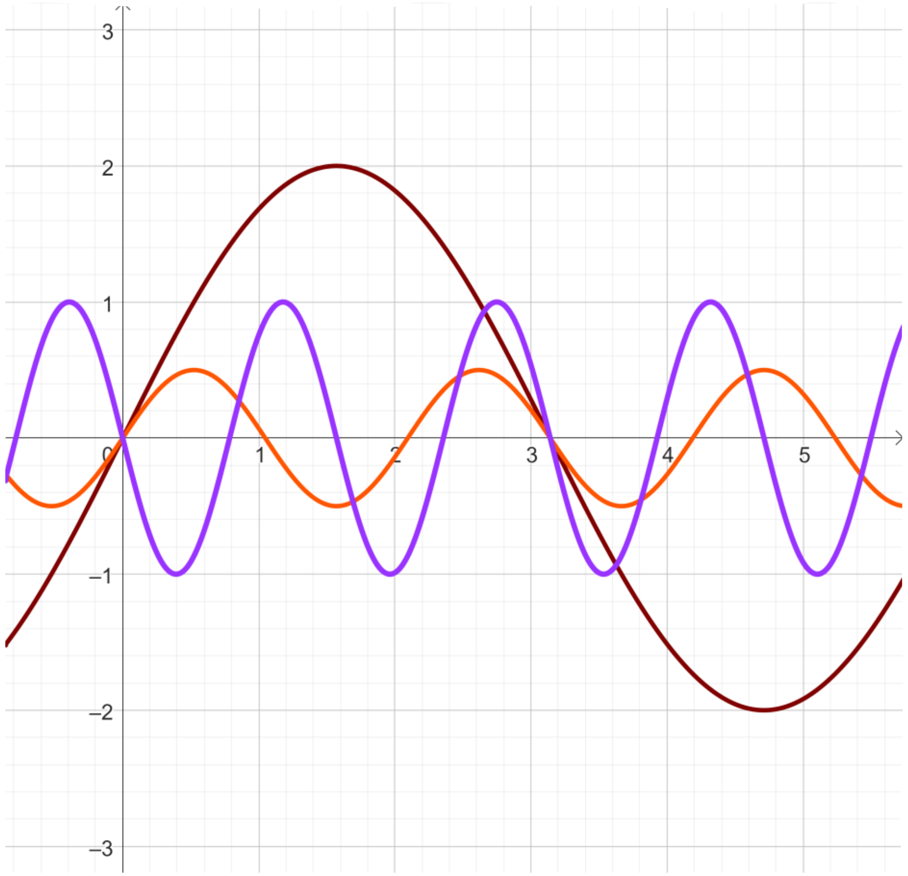
Allgemeine Formel der Fourier Reihe

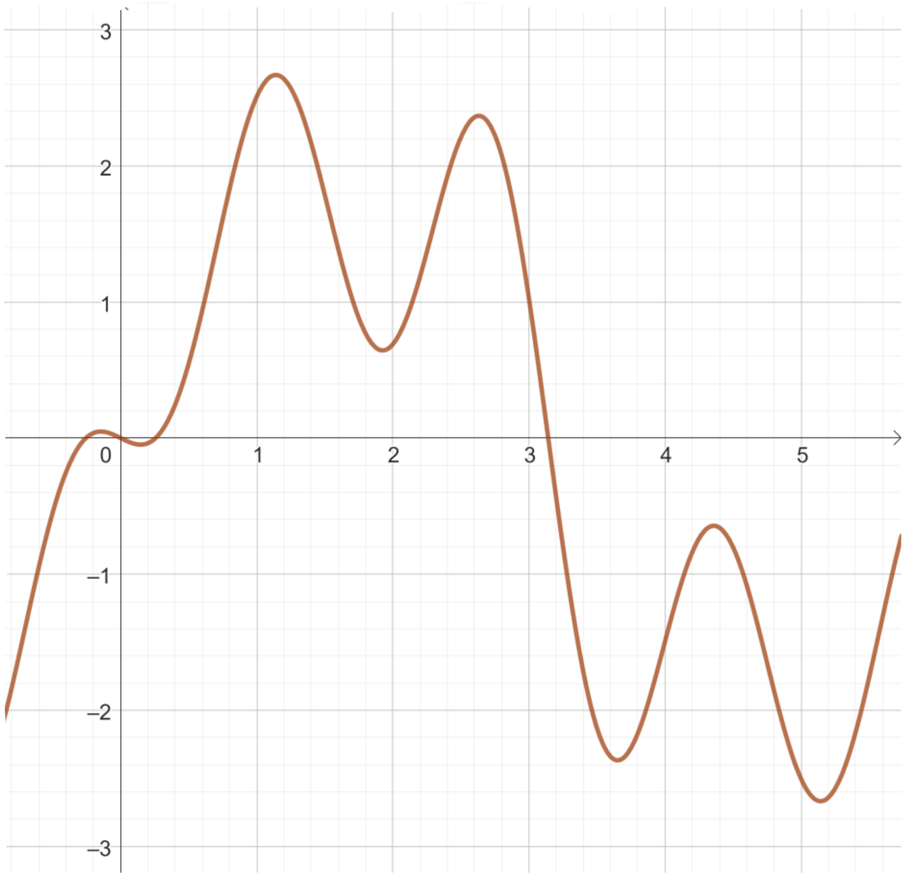
$$f(x) = \frac{A_0}{2} + \sum_{k=0}^{\infty} \left(A_k \cdot cos(x \cdot \omega_k) + B_k \cdot sin(x \cdot \omega_k) \right)$$

Teil 2 – Fourier Transformation

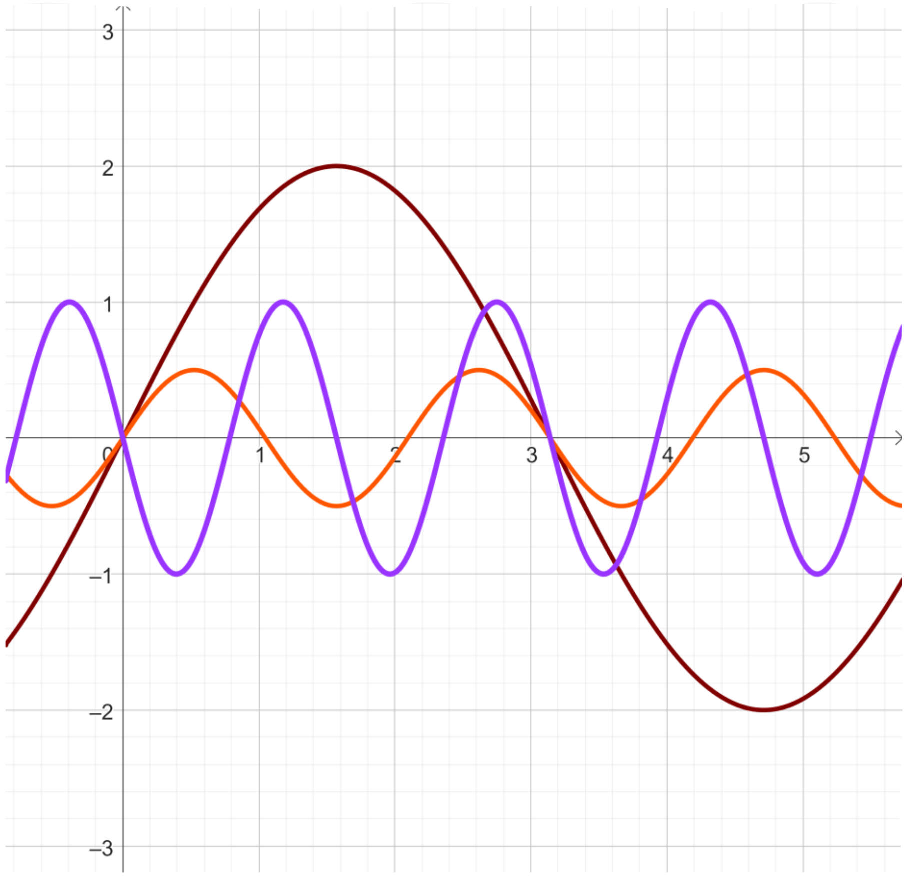
Mathematische Darstellung eigener Signale

Frequenz- und Zeitfunktion





Frequenz- und Zeitfunktion

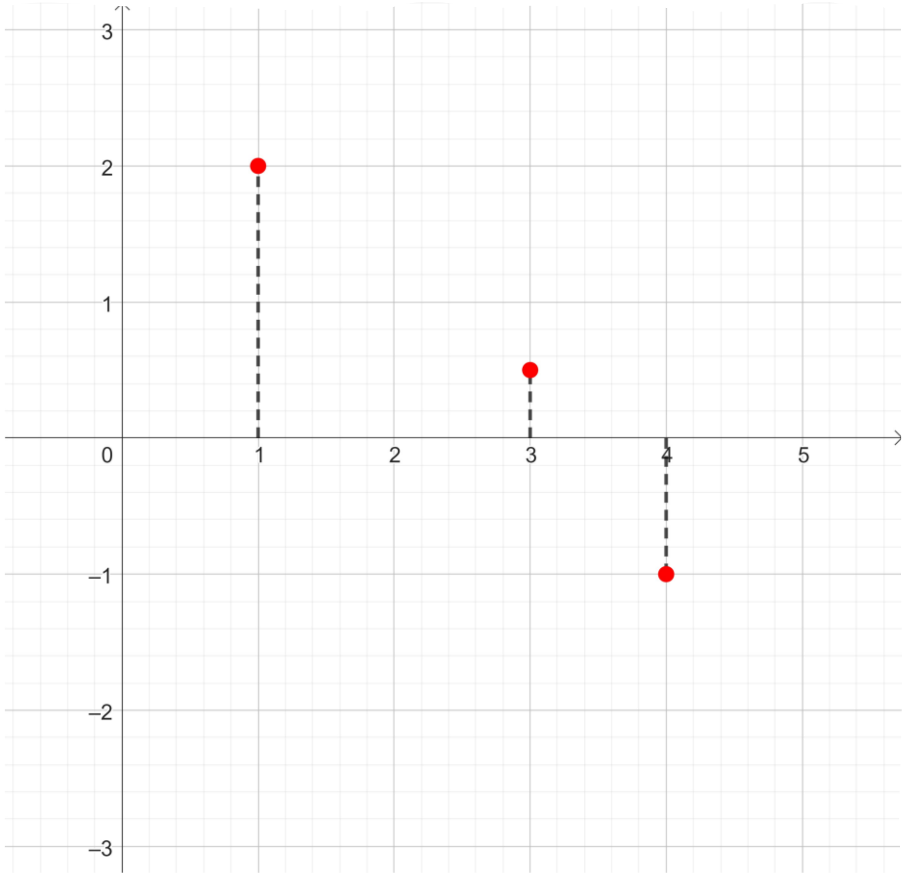


$$f(x) = 2 \cdot sin(x)$$

$$f(x) = 0{,}5 \cdot sin(3x)$$

$$f(x) = -sin(4x)$$

Frequenz- und Zeitfunktion



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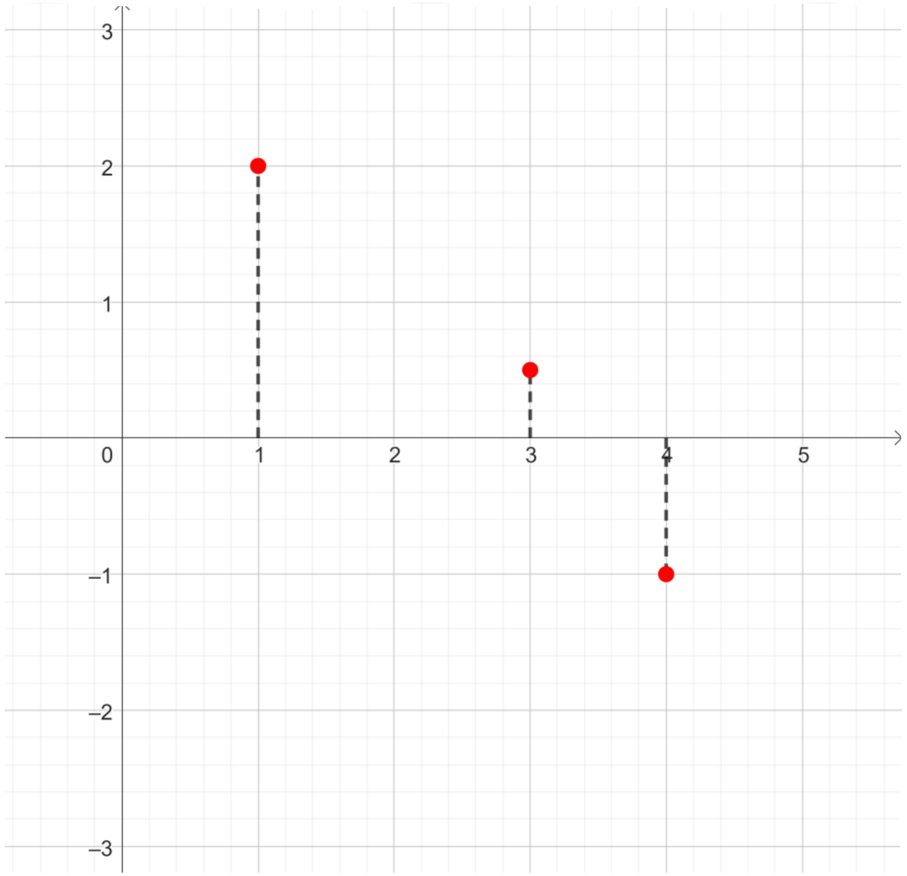
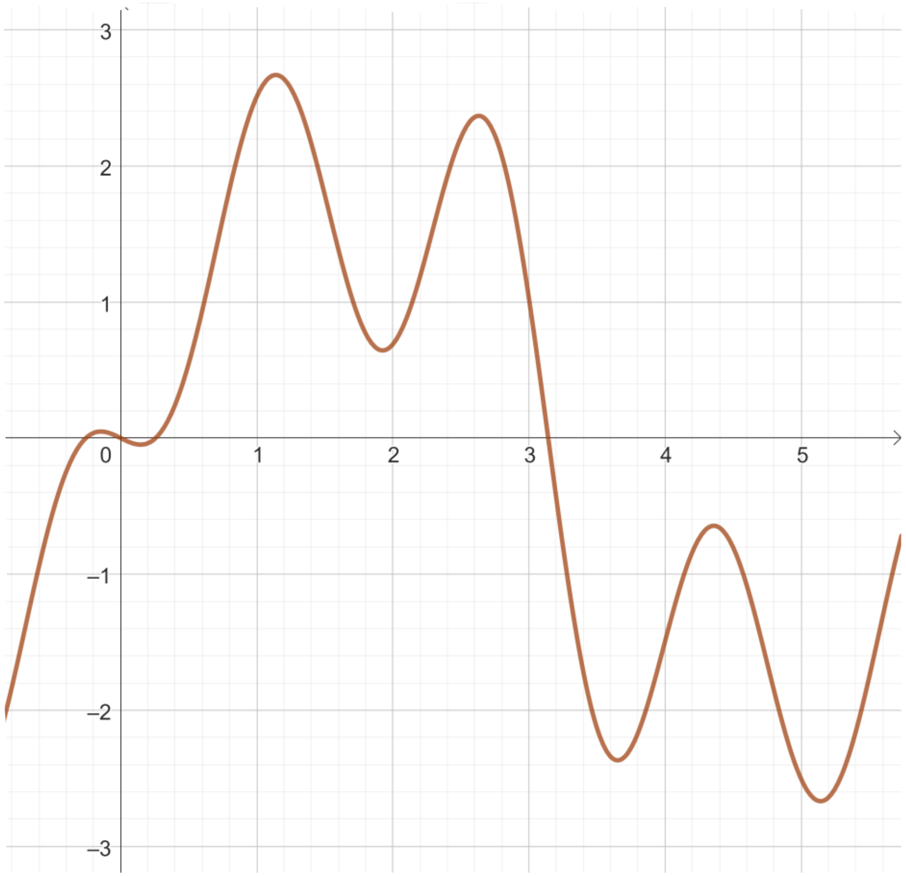
Frequenz- und Zeitfunktion





Herleitung Fourier Transformation

$$F(\omega) = \int_{-\infty}^{\infty} \left(f(t) e^{-i \cdot \omega \cdot t} \right) dt$$

Herleitung Fourier Transformation

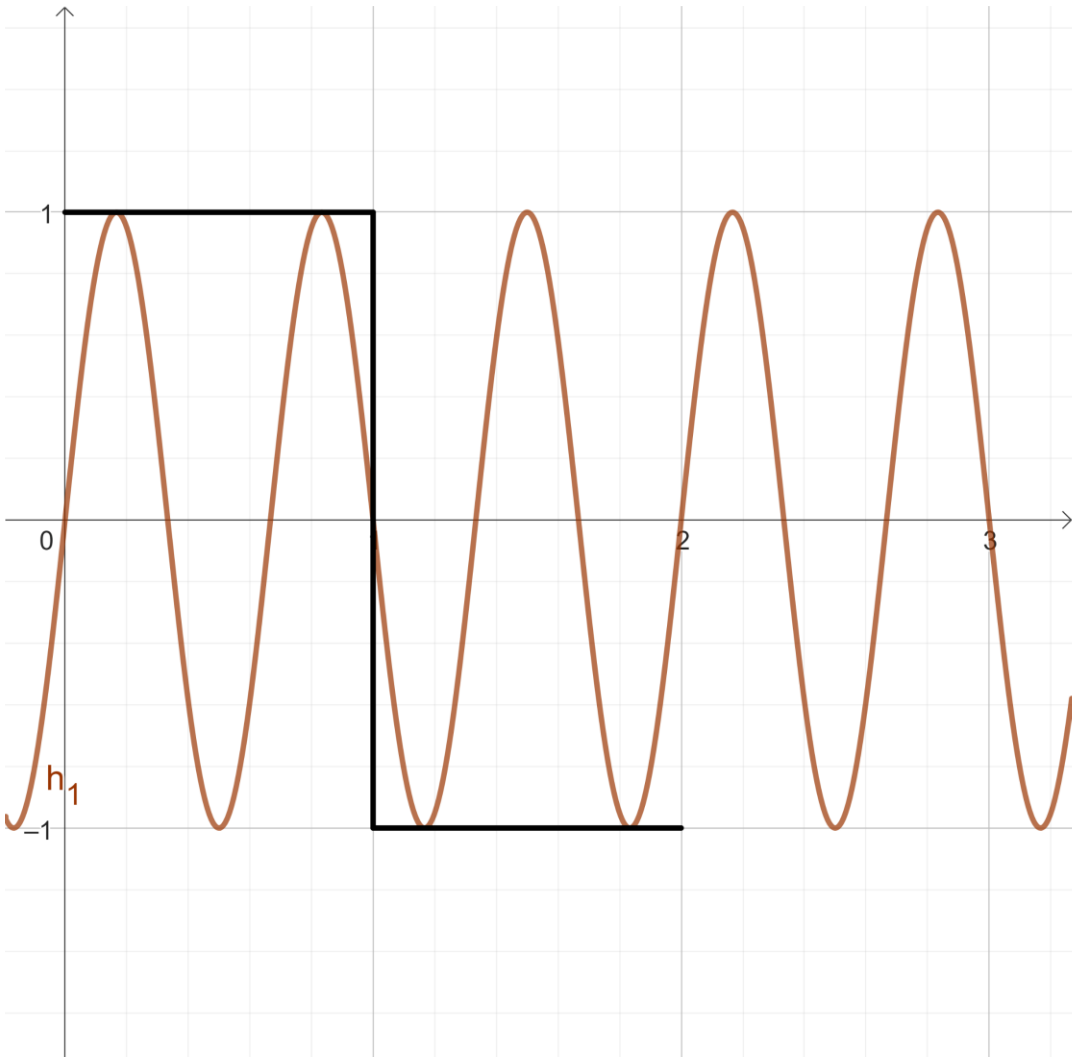
$$F(\omega) = \int_{-\infty}^{\infty} \left(f(t)\boldsymbol{e}^{-\boldsymbol{i}\cdot\boldsymbol{\omega}\cdot\boldsymbol{t}}\right) dt$$

$$F(\omega) = \int_{-\infty}^{\infty} \left(f(t)\left(\boldsymbol{cos}(\boldsymbol{\omega t}) - \boldsymbol{i} \cdot \boldsymbol{sin}(\boldsymbol{\omega t})\right)\right) dt$$

Herleitung am Beispiel

$$F(\omega) = \int_{-\infty}^{\infty} \Big(f(t)\big(cos(\omega t) - i \cdot sin(\omega t)\big)\Big)\, dt$$

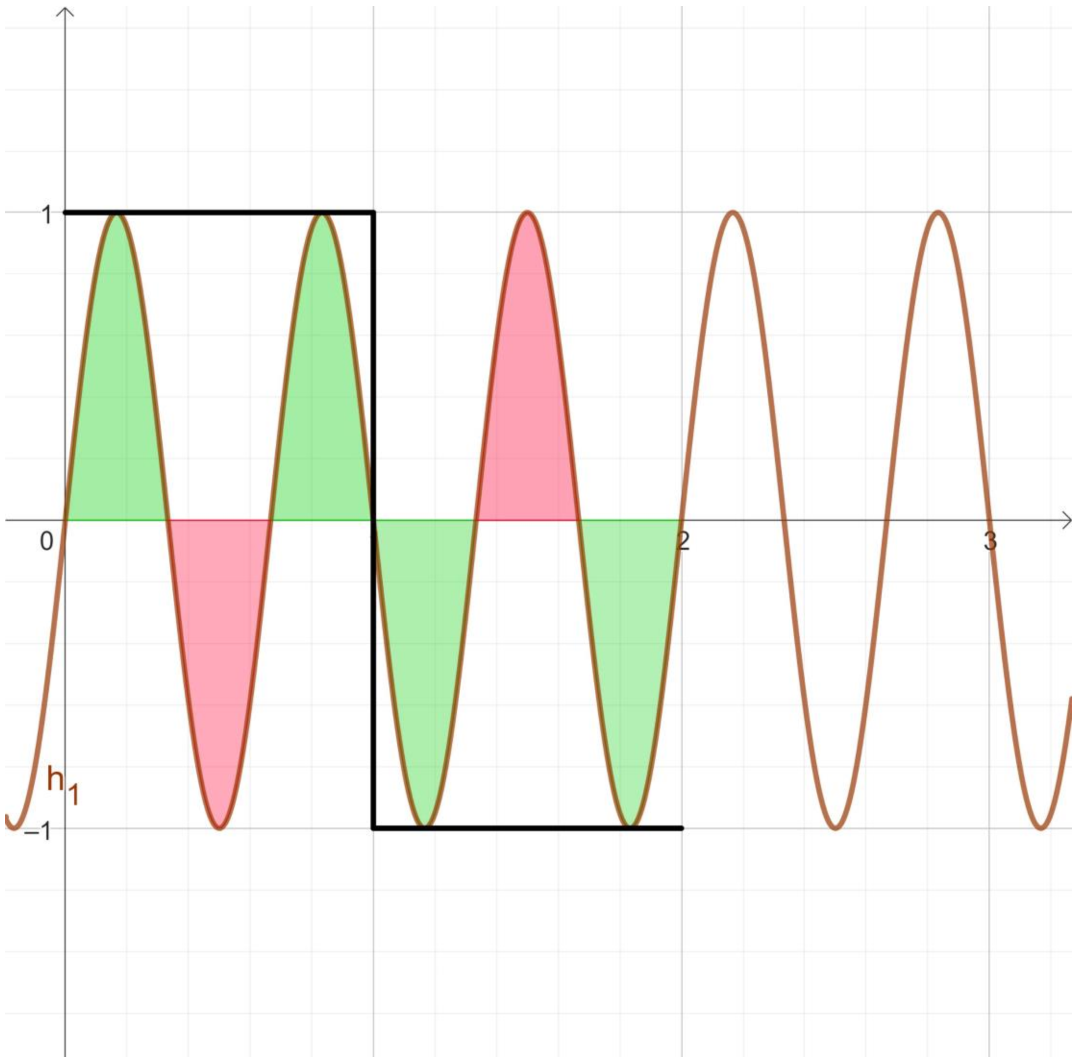
$$F(\mathbf{3\pi}) = \int_{-\infty}^{\infty} \Big(f(t)\big(cos(\mathbf{3\pi t}) - i \cdot sin(\mathbf{3\pi t})\big)\Big)\, dt$$



Herleitung am Beispiel

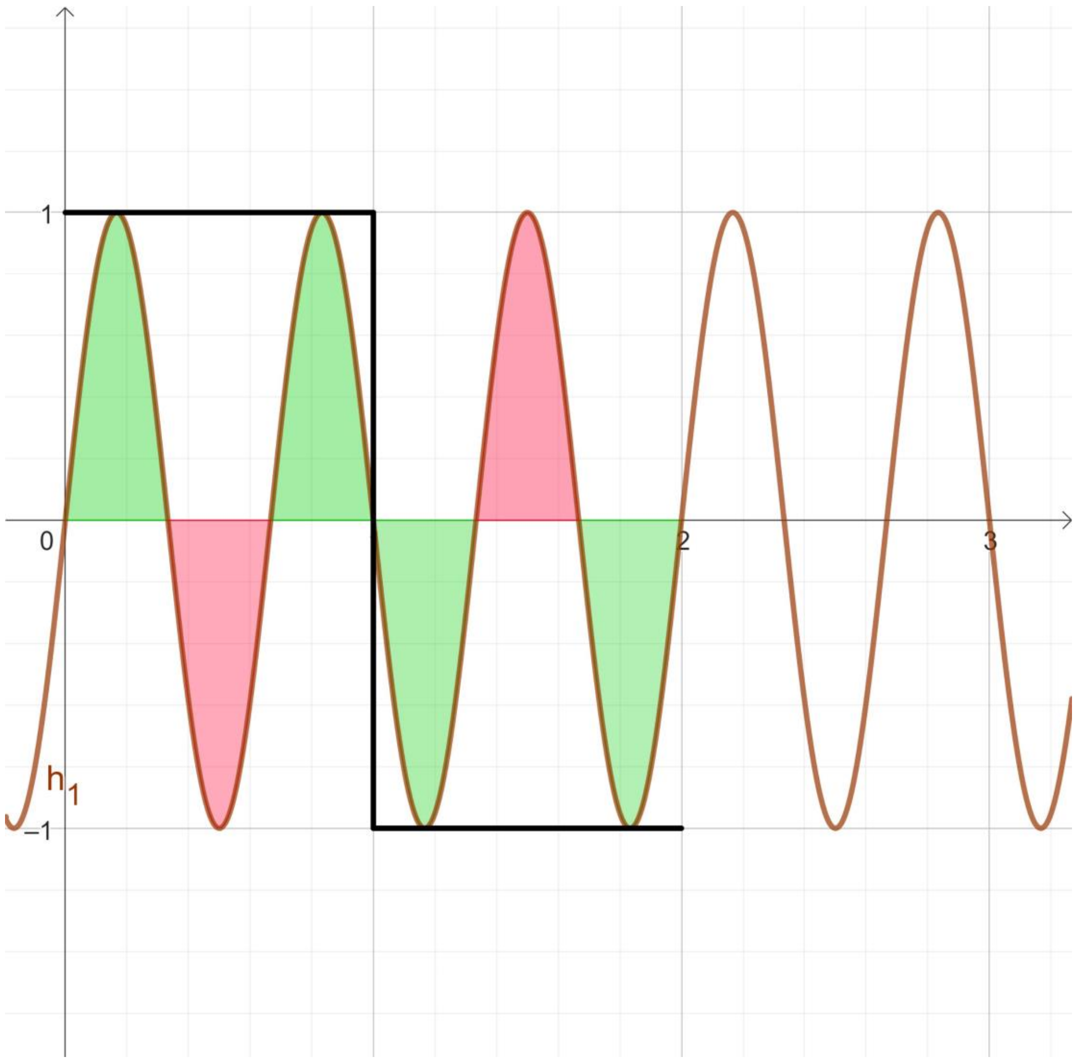
$$F(3\pi) = \int_{-\infty}^{\infty} \Big(f(t)\big(cos(3\pi t) - i \cdot sin(3\pi t)\big)\Big)\, dt$$

$$F(3\pi) = \int_{-\infty}^{\infty} \big(f(t) \cdot sin(3\pi t)\big) dt$$



Herleitung am Beispiel

$$F(3\pi) = 0{,}45345345 = \frac{4}{\pi} \cdot \frac{1}{3}$$



Skalierung der Fourier Transformation

$$\frac{\mathbf{1}}{\sqrt{\boldsymbol{T^n}}}$$

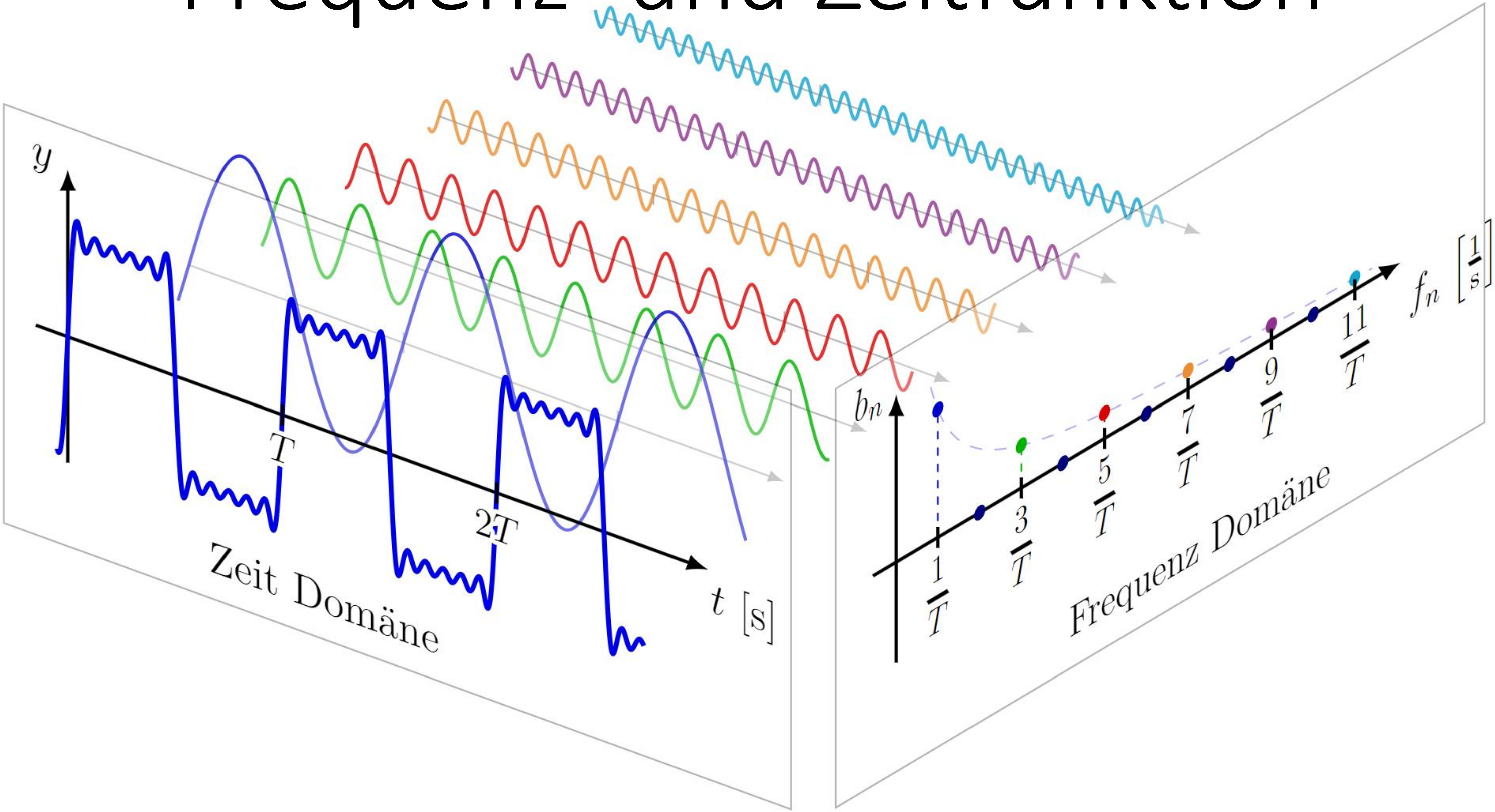
Inverse Fourier Transformation:

$$f(t) = \frac{\mathbf{1}}{\mathbf{2\pi}} \int_{-\infty}^{\infty} F(\omega) e^{i\cdot\omega\cdot t} d$$

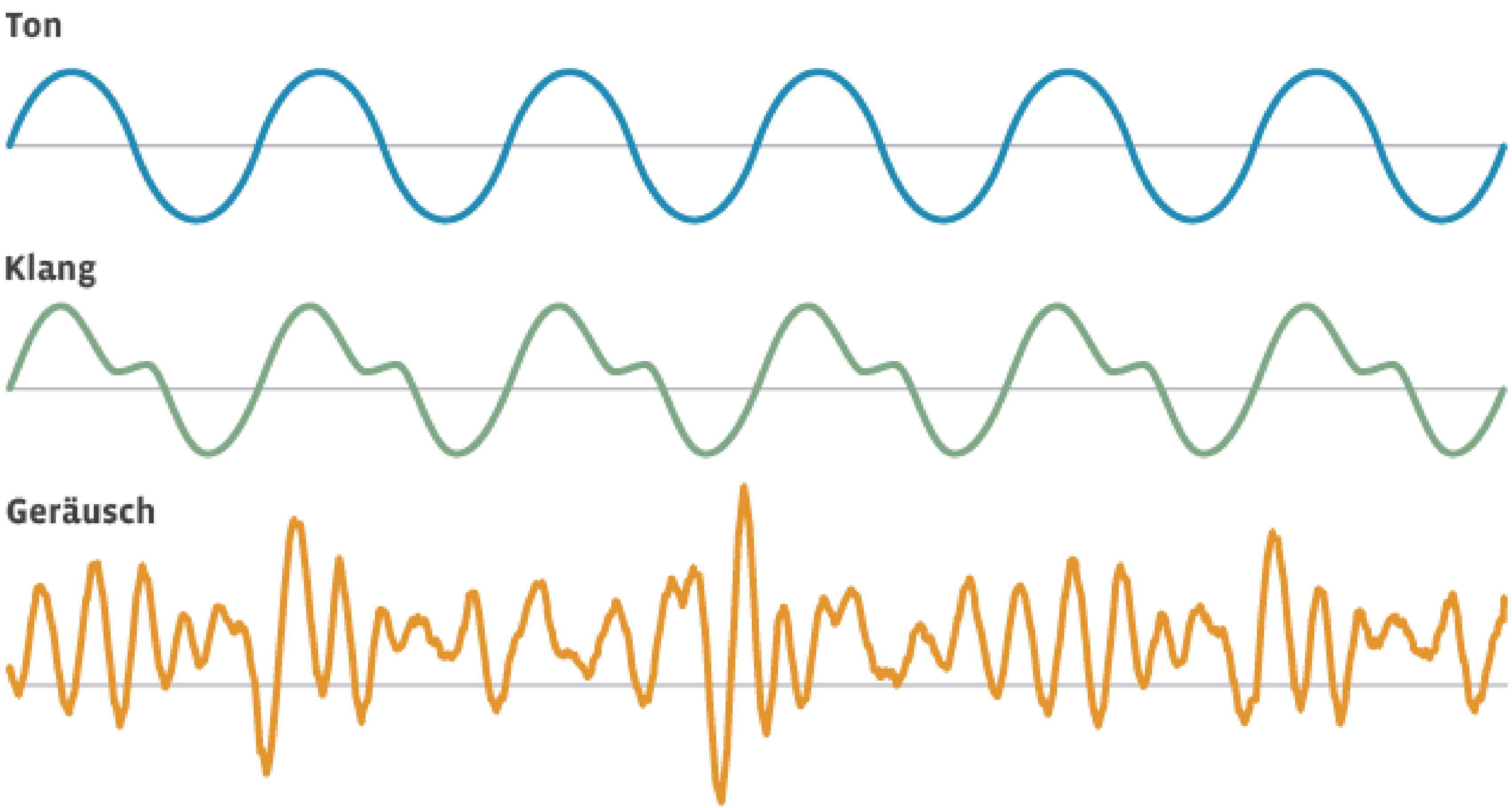
Fourier Transformation:

$$F(\omega) = \int_{-\infty}^{\infty} \left(f(t) e^{-i\cdot\omega\cdot t}\right) dt$$

Frequenz- und Zeitfunktion



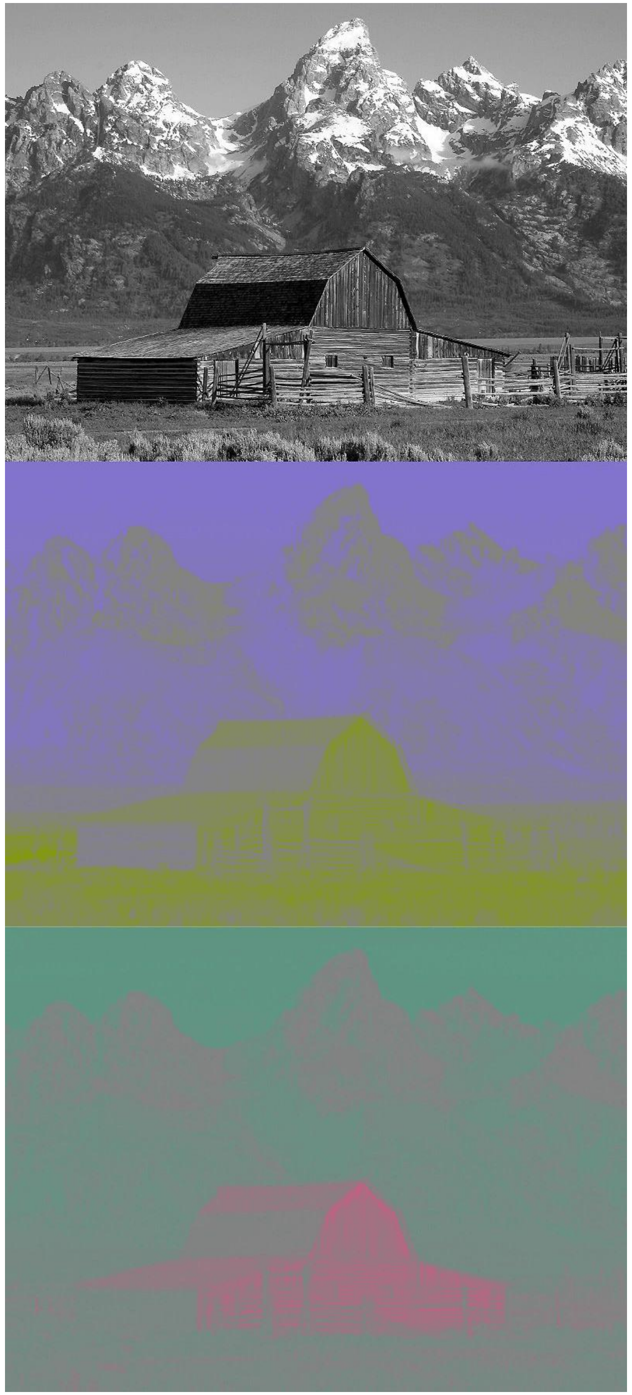
Anwendungsbezug



Teil 3 – Anwendungsbeispiel JPEG

Funktionsweise und Beispiel

Farbraumumwandlung und Chrominanz-Vereinfachung



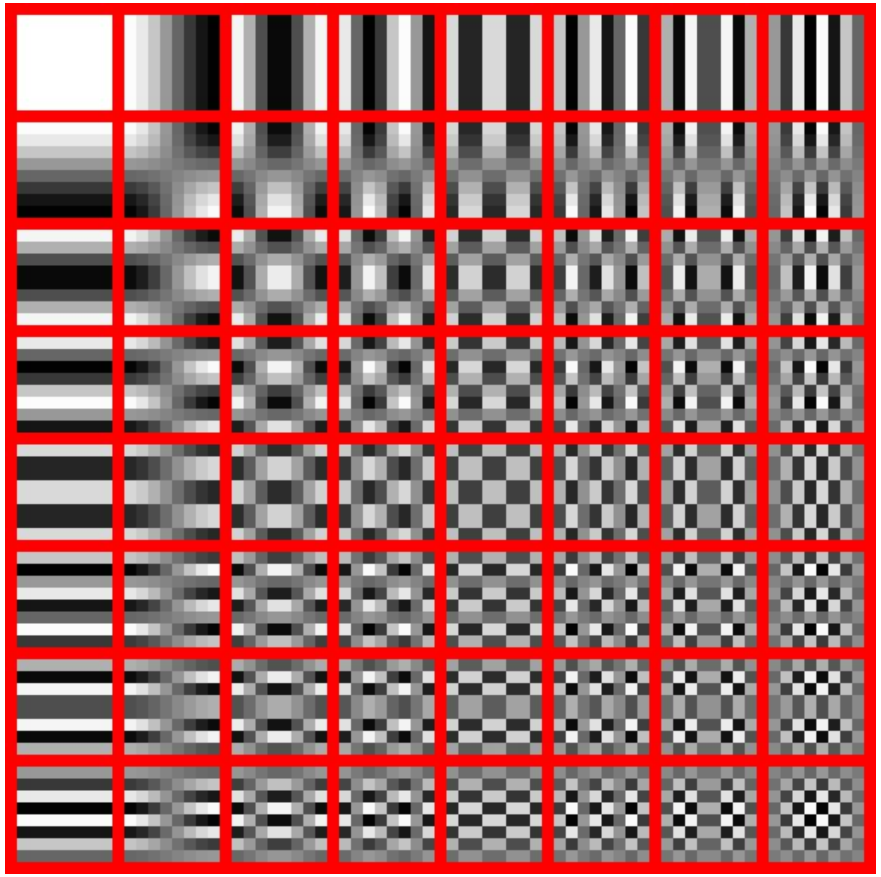
Diskrete-Kosinus-Transformation

37	36	37	42	47	50	55	61
37	37	38	44	49	52	54	57
38	39	44	52	56	57	56	56
37	44	56	65	68	67	62	59
43	57	72	83	85	80	73	66
54	57	97	105	102	93	84	77
72	101	117	116	111	105	95	85
91	115	124	121	116	108	100	92

Diskrete-Kosinus-Transformation

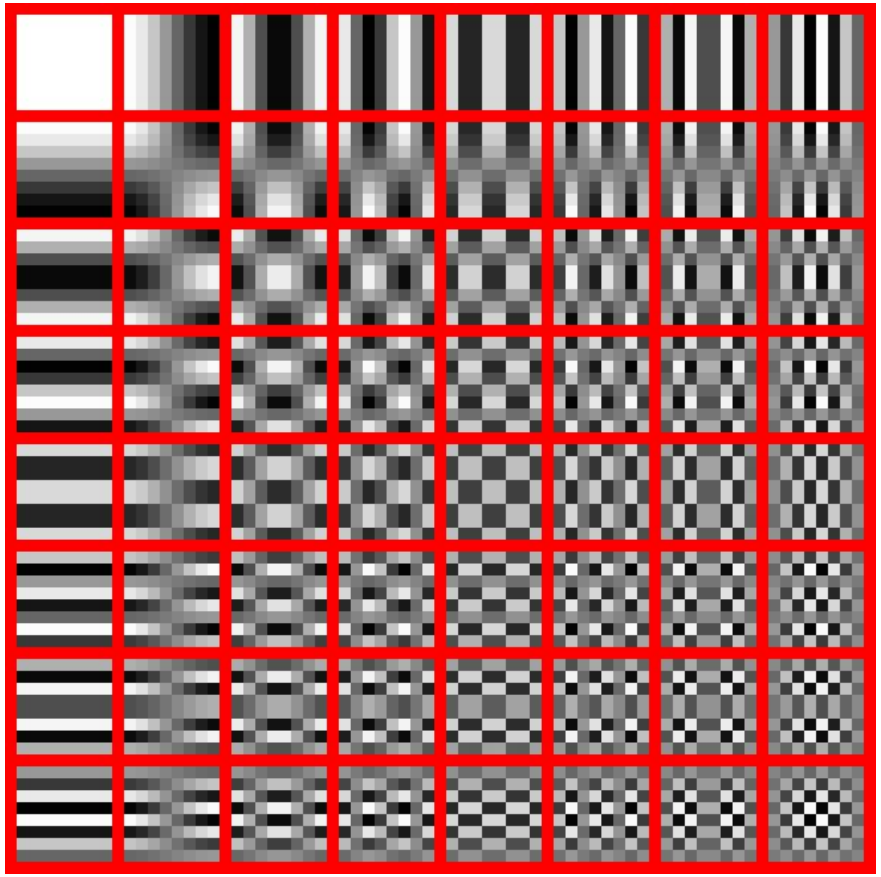
-91	-92	-91	-86	-81	-78	-73	-67
-91	-91	-90	-84	-79	-76	-74	-71
-90	-89	-84	-76	-72	-71	-72	-72
-91	-84	-72	-63	-60	-61	-66	-69
-85	-71	-56	-45	-43	-48	-55	-62
-74	-71	-31	-23	-26	-35	-44	-51
-56	-27	-11	-12	-17	-23	-33	-43
-37	-13	-4	-7	-12	-20	-28	-36

Diskrete-Kosinus-Transformation



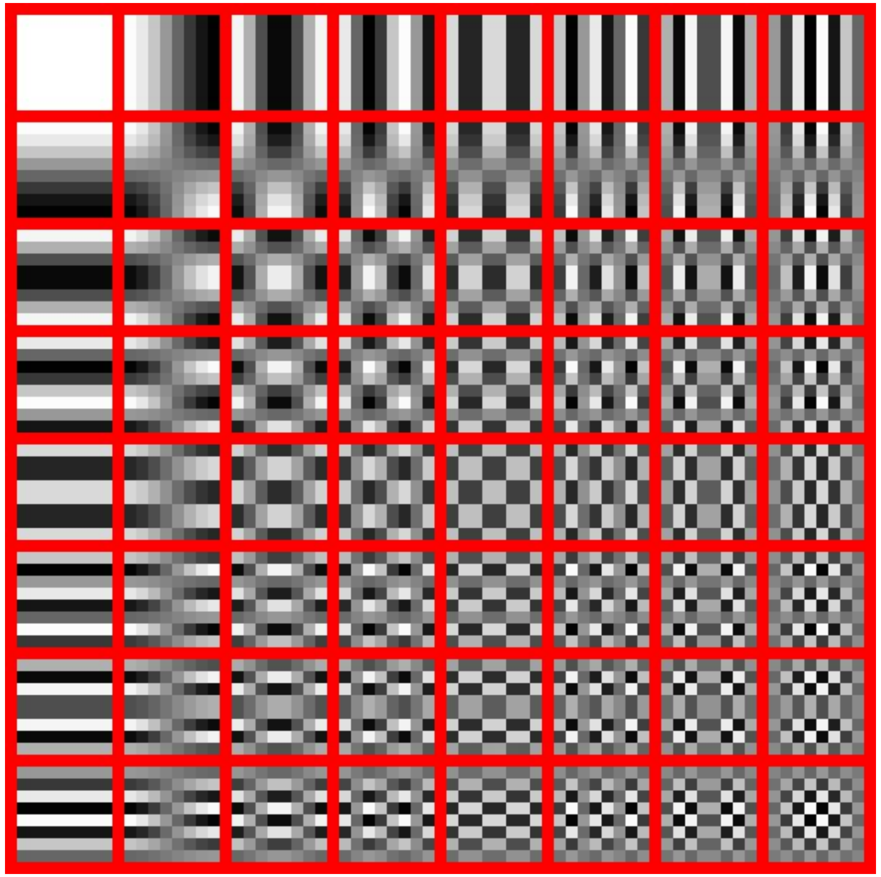
Diskrete-Kosinus-Transformation

560	-41	-57	-12	-4	0	1	1
-180	-25	43	14	12	0	0	-1
42	17	16	-3	-4	-4	-2	0
-1	-12	-11	-3	3	4	4	2
1	2	4	1	-1	-3	-3	-1
1	0	-1	0	5	4	4	2
-2	-3	-1	0	5	4	4	2
2	3	2	0	-3	-4	-4	-2



Quantisierung

4	3	4	4	4	6	11	15
3	3	3	4	5	8	14	19
3	4	4	5	8	12	16	20
4	5	6	7	12	14	18	20
6	6	9	11	14	17	21	23
9	12	12	18	23	22	25	21
11	13	15	17	21	23	25	21
13	12	12	13	16	19	21	21



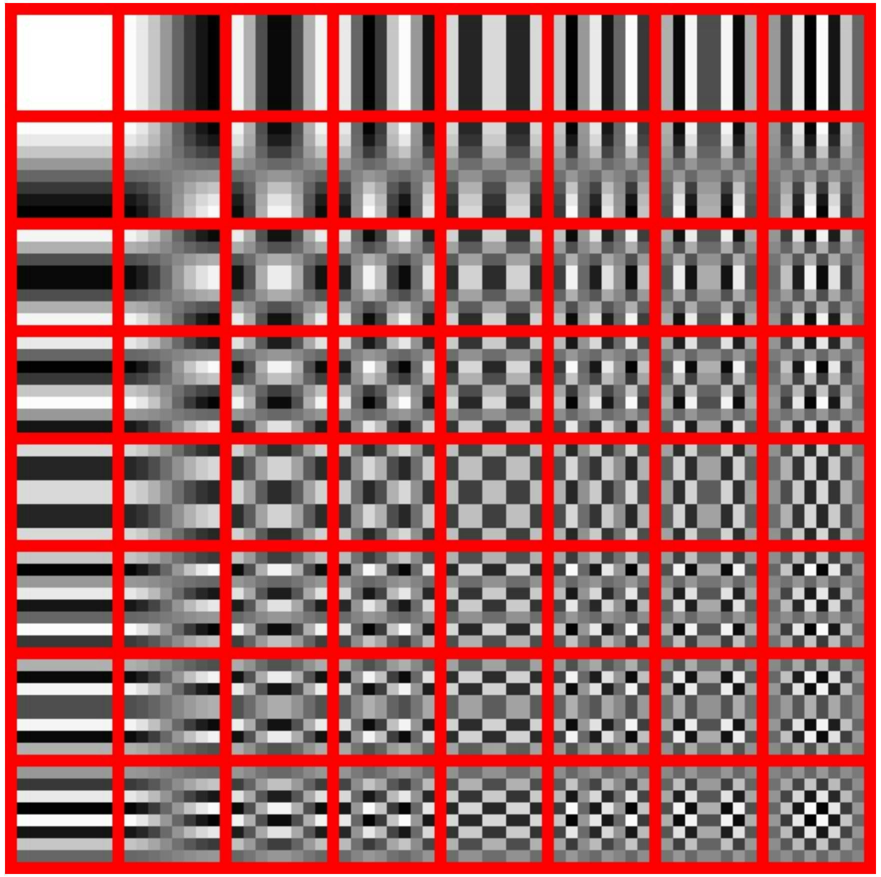
Quantisierung

4	3	4	4	4	6	11	15
3	3	3	4	5	8	14	19
3	4	4	5	8	12	16	20
4	5	6	7	12	14	18	20
6	6	9	11	14	17	21	23
9	12	12	18	23	22	25	21
11	13	15	17	21	23	25	21
13	12	12	13	16	19	21	21

560	-41	-57	-12	-4	0	1	1
-180	-25	43	14	12	0	0	-1
42	17	16	-3	-4	-4	-2	0
-1	-12	-11	-3	3	4	4	2
1	2	4	1	-1	-3	-3	-1
1	0	-1	0	5	4	4	2
-2	-3	-1	0	5	4	4	2
2	3	2	0	-3	-4	-4	-2

Quantisierung

140	-14	-14	-3	-1	0	0	0
-60	-8	14	4	2	0	0	0
14	4	4	-1	-1	0	0	0
0	-2	-2	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



Huffman- und Lauflängenkodierung

140 -14 -60 14 -8 -14 -3 14 4 0 0 -2 4
4 -1 0 2 -1 -2 0 0 0 0 0 0 -1 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0

140 -14 -60 14 -8 -14 -3 14 4 **2x** 0 -2
2x 4 -1 0 2 -1 -2 **6x** -1 **38x** 0

Fazit

Persönliches Fazit und Zusammenfassung